

## Lecture: Retention Policies

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Discounted Cash Flow, Section 3.3



# Outline

## 3.3.1 Autonomous retention

Definition and valuation

Finite example

## 3.3.2 Retention based on cash flows

Definition and valuation

Infinite lifetime

Finite example

## 3.3.3 Retention based on dividends

Definition and valuation

Finite example

## 3.3.4 Retention based on market values

Definition and valuation

Finite example



We analyse four different retention policies:

- autonomous retention,
- retention based on cash flows,
- retention based on dividends,
- retention based on market values.



**Definition:** A firm follows autonomous retention if  $\tilde{A}_t$  is deterministic.

**Valuation:** The following equations holds

$$\tilde{V}_t^I = \tilde{V}_t^u + (1 - \tau^D) A_t + \frac{\tau^I (1 - \tau^D) r_f A_t}{1 + r_f (1 - \tau^I)} + \dots + \frac{\tau^I (1 - \tau^D) r_f A_{T-1}}{(1 + r_f (1 - \tau^I))^{T-t}}.$$

If, in particular, retention is constant and the firm lives on for ever, then

$$\tilde{V}_t^I = \tilde{V}_t^u + \frac{1 - \tau^D}{1 - \tau^I} A.$$



Let us assume that

$$A_0 = 10, \quad A_1 = 20, \quad A_2 = 0.$$

With that we get

$$\begin{aligned} V_0^l &= V_0^u + \tilde{V}_t^u + (1 - \tau)A_0 + \frac{\tau(1 - \tau)r_f A_0}{1 + r_f(1 - \tau)} + \frac{\tau(1 - \tau)r_f A_1}{(1 + r_f(1 - \tau))^2} \\ &= 249.692 + (1 - 0.5) \times 10 + \frac{0.5 \times (1 - 0.5) \times 0.1 \times 10}{1 + 0.1 \times (1 - 0.5)} \\ &\quad + \frac{0.5 \times (1 - 0.5) \times 0.1 \times 20}{(1 + 0.1 \times (1 - 0.5))^2} \approx 255.383 \end{aligned}$$

for the value of the levered firm.



**Definition:** Retention is based on cash flows if the firm retains a determined percentage of cash flows

$$\tilde{A}_t = \alpha_t \widetilde{FCF}_t^u.$$

**Valuation:** The following equations holds

$$\begin{aligned} \tilde{V}_t^l = & \tilde{V}_t^u + \frac{(1+r_f)(1-\tau^D)\alpha_t \widetilde{FCF}_t^u}{1+r_f(1-\tau^l)} \\ & + \frac{\tau^l r_f (1-\tau^D)}{1+r_f(1-\tau^l)} \left( \frac{E[\alpha_{t+1} \widetilde{FCF}_{t+1}^u | \mathcal{F}_t]}{1+k^{E,u}} + \dots + \frac{E[\alpha_{T-1} \widetilde{FCF}_{T-1}^u | \mathcal{F}_t]}{(1+k^{E,u})^{T-t}} \right). \end{aligned}$$



Quite straightforward:

$$\begin{aligned}\tilde{V}_t^I &= \tilde{V}_t^u + (1-\tau^D)\tilde{A}_t + \frac{\mathbb{E}_Q[\tau^I r_f (1-\tau^D)\tilde{A}_t | \mathcal{F}_t]}{1+r_f(1-\tau^I)} + \dots + \frac{\mathbb{E}_Q[\tau^I r_f (1-\tau^D)\tilde{A}_{T-1} | \mathcal{F}_t]}{(1+r_f(1-\tau^I))^{T-t}} \\ &= \tilde{V}_t^u + (1-\tau^D)\alpha_t \widetilde{FCF}_t^u + \frac{\tau^I r_f (1-\tau^D)\alpha_t \widetilde{FCF}_t^u}{1+r_f(1-\tau^I)} \\ &\quad + \frac{\tau^I r_f (1-\tau^D)}{1+r_f(1-\tau^I)} \left( \frac{\mathbb{E}_Q[\alpha_{t+1} \widetilde{FCF}_{t+1}^u | \mathcal{F}_t]}{1+r_f(1-\tau^I)} + \dots + \frac{\mathbb{E}_Q[\alpha_{T-1} \widetilde{FCF}_{T-1}^u | \mathcal{F}_t]}{(1+r_f(1-\tau^I))^{T-t}} \right)\end{aligned}$$

and use Theorem 3.4.



If the firm lives for ever and  $\alpha$  is constant, we get

$$\tilde{V}_t^l = \left( 1 + \frac{\tau^l r_f (1 - \tau^D) \alpha}{1 + r_f (1 - \tau^l)} \right) \tilde{V}_t^u + \frac{(1 + r_f) (1 - \tau^D) \alpha}{1 + r_f (1 - \tau^l)} \widetilde{FCF}_t^u.$$



We assume

$$\alpha_0 = 0\%, \quad \alpha_1 = 10\%, \quad \alpha_2 = 20\%.$$

With that we get

$$\begin{aligned} V_0^l &= V_0^u + \frac{(1+r_f)(1-\tau)\alpha_0 FCF_0^u}{1+r_f(1-\tau)} + \frac{\tau r_f(1-\tau)}{1+r_f(1-\tau)} \left( \alpha_1 \frac{E[\widetilde{FCF}_1^u]}{1+kE,u} \right. \\ &\quad \left. + \alpha_2 \frac{E[\widetilde{FCF}_2^u]}{(1+kE,u)^2} \right) \\ &= 249.692 + 0 + \frac{0.5 \times 0.1 \times (1-0.5)}{1+0.1 \times (1-0.5)} \times \left( 0.1 \times \frac{100}{1+0.15} + 0.2 \times \frac{110}{(1+0.15)^2} \right) \\ &\approx 250.295 \end{aligned}$$

for the value of the levered firm.



Retention is based on dividends if until  $n \ll T$  the dividend is determined. We furthermore assume that the investment policy is given and that the free cash flows suffice to pay this dividend. Hence, the retention has to be

$$\tilde{A}_t = \left( \frac{1}{1 - \tau^D} \widetilde{FCF}_t^u + (1 + \tilde{r}_{t-1})\tilde{A}_{t-1} - Div_t \right)^+$$

for all  $t = 1, \dots, n$ .

Because of the maximum function **without any further assumptions** a valuation equation has to take **options into account**. If we assume that the **retention stays positive** or

$$\frac{1}{1 - \tau^D} \widetilde{FCF}_t^u \geq Div_t \quad \forall t \leq n$$

then a closed form solution will be possible.



**Valuation:** *The valuation equation then reads*

$$\begin{aligned} \tilde{V}_U^I &= \tilde{V}_t^u + \tau^I (1 - \tau^D) \left( \frac{1 + r_f}{1 + r_f(1 - \tau^I)} \right)^{n-t+1} \tilde{A}_t \\ &+ \tau^I r_f \sum_{v=t+1}^n \left( \frac{E[\tilde{FCF}_v^u | \mathcal{F}_t]}{(1 + k^{E,u})^{v-t}} - \frac{(1 - \tau^D) \text{Div}_v}{(1 + r_f(1 - \tau^I))^{v-t}} \right) \left( 1 + \left( \frac{1 + r_f}{1 + r_f(1 - \tau^I)} \right)^{n+1-v} \right). \end{aligned}$$

We consider generalization for  $n \rightarrow \infty$  as being useless (then the formula might violate the transversality condition).



Let us assume

$$A_0 = 0, \quad Div_1 = Div_2 = 40.$$

If the tax rate amounts to 50%, cash flows are greater than the dividend. Under these conditions we get

$$V_0^l = V_0^u + \tau r_f \left( \frac{E[\widetilde{FCF}_1^u]}{1+kE,u} - \frac{(1-\tau)Div_1}{1+(1-\tau)r_f} \right) \left( 1 + \left( \frac{1+r_f}{1+(1-\tau)r_f} \right)^2 \right) + \\ + \tau r_f \left( \frac{E[\widetilde{FCF}_2^u]}{(1+kE,u)^2} - \frac{(1-\tau)Div_2}{(1+(1-\tau)r_f)^2} \right) \left( 1 + \frac{1+r_f}{1+(1-\tau)r_f} \right)$$

and from that

$$V_0^l = 249.692 + 0.1 \times 0.5 \times \left( \frac{100}{1+0.15} - \frac{(1-0.5) \times 40}{1+(1-0.5) \times 0.1} \right) \times \left( 1 + \left( \frac{1+0.1}{1+(1-0.5) \times 0.1} \right)^2 \right) + \\ + 0.1 \times 0.5 \times \left( \frac{110}{(1+0.15)^2} - \frac{(1-0.5) \times 40}{(1+(1-0.5) \times 0.1)^2} \right) \times \left( 1 + \frac{1+0.1}{1+(1-0.5) \times 0.1} \right) V_0^l \approx 263.472$$

for the levered firm.



Managers now retain a fixed proportion of levered firm value. The more valuable the firm, the more managers retain. We assume that the factor

$$\tilde{l}_t = \frac{\tilde{A}_t}{\tilde{V}_t^l}$$

is deterministic.

'Growth financed by constant intensity'



**Valuation:** *Valuation equation reads*

$$\tilde{V}_t^I = \sum_{s=t+1}^T \frac{\mathbb{E} \left[ \prod_{h=t+1}^{s-1} (1 - (1 - \tau^D) l_h) \widetilde{FCF}_s^u | \mathcal{F}_t \right]}{\prod_{h=t}^{s-1} (1 + k_h)},$$

whereby

$$1 + k_h = (1 + k^{E,u}) \left( 1 - \frac{(1 + r_f)(1 - \tau^D)}{1 + r_f(1 - \tau^I)} l_h \right).$$

This equation is very similar to the Miles-Ezzell formula.



We assume a retention–value ratio of

$$l_0 = l_1 = l_2 = 10\%.$$

The following then results for the cost of equity:

$$\begin{aligned} k_t &= (1 + k_t^{E,u}) \left( 1 - \frac{(1+r_f)(1-\tau)}{1+r_f(1-\tau)} l_h \right) - 1 \\ &= (1+0.15) \cdot \left( 1 - \frac{(1+0.1) \times (1-0.5)}{1+0.1 \times (1-0.5)} \times 0.1 \right) - 1 \approx 8.976\% . \end{aligned}$$

With that we get the value of the levered firm as

$$\begin{aligned} V_0^l &= \frac{E[\widetilde{FCF}_1^u]}{1+k} + \frac{(1-(1-\tau)l_1) E[\widetilde{FCF}_2^u]}{(1+k)^2} + \frac{(1-(1-\tau)l_1)(1-(1-\tau)l_2) E[\widetilde{FCF}_3^u]}{(1+k)^3} \\ &\approx \frac{100}{1+0.08976} + \frac{(1-(1-0.5) \times 0.1)110}{(1+0.08976)^2} + \frac{(1-(1-0.5) \times 0.1)^2 \times 121}{(1+0.08976)^3} \approx 264.137 . \end{aligned}$$



We considered four different retention policies and their impact on the firm value. These were

- autonomous retention,
- retention based on cash flows,
- retention based on dividends,
- retention based on market values.

